

the fundamental, steady, and second-harmonic components into which the alternating plate current i_p may be resolved.

3-20. Series Expansion for Alternating Plate Current.—Theoretically, it should be possible to predict from Eq. (3-2) and corresponding tetrode and pentode equations the form of the alternating plate current corresponding to an exciting voltage of known wave form. Practically, however, $F(e_b + \mu e_c + e)$ is so complicated in form that Eq. (3-2) is of little or no value for this purpose. The behavior of a nonlinear circuit element can in general be analyzed mathematically most readily by expressing the alternating current in the form of an infinite power series. For a two-terminal element, in which the current depends upon only one voltage, the series involves only the impressed voltage, the element impedance and its derivatives at the operating point, and the circuit impedance. It may be derived by the application of Taylor's expansion for a function of one variable to the functional equation of current. Because the plate current of a vacuum tube depends upon all the electrode voltages, a complete series expansion applicable to a tube with three or more electrodes must involve all electrode voltages, as well as various tube factors and their derivatives at the operating point.¹ The general series expansion for tetrodes and pentodes is complicated, therefore, particularly if control-grid current is assumed to flow and the control-grid circuit contains an impedance. When all electrode voltages except those of the control grid and the plate are constant, however, as is usually true, the action of multigrid tubes is similar to that of triodes, and the general form of the series reduces to that for a triode. The triode expansion may consequently be applied to any tube in which only the plate and control-grid voltages vary. Furthermore, most problems in which the series expansion is of value can be adequately treated by the use of the form derived under the assumption that control-grid current does not flow. A further simplification results from the assumption that μ is constant, which is approximately true in many tubes. For the general forms of the series expansion for plate current of multigrid tubes, and for the similar expansions for currents to other electrodes, the student should refer to the work of Llewellyn² and others.³

3-21. Series Expansion. Resistance Load.—The series expansion for the plate current of a triode with negative grid voltage, constant amplification factor, and nonreactive load has the following form:

$$i_p = a_1 e + a_2 e^2 + a_3 e^3 + \dots \quad (3-40)$$

¹ CARSON, J. R., *Proc. I.R.E.*, **7**, 187 (1919).

² LLEWELLYN, F. B., *Bell System Tech. J.*, **5**, 433 (1926).

³ BRAINERD, J. G., *Proc. I.R.E.*, **17**, 1006 (1929); CAPOREALE, P., *Proc. I.R.E.*, **18**, 1593 (1930); BONER, M. O., *Phys. Rev.*, **39**, 863 (1932); BENNETT, W. R., *Bell System Tech. J.*, **12**, 228 (1933); ESPLEY, D. C., *Proc. I.R.E.*, **22**, 781 (1934); BARROW, W. L., *Proc. I.R.E.*, **22**, 964 (1934).

where

$$\left. \begin{aligned} a_1 &= \frac{\mu}{r_p + r_b} \\ a_2 &= -\frac{\mu^2 r_p}{2(r_p + r_b)^3} \frac{\partial r_p}{\partial e_b} \\ a_3 &= \frac{\mu^3 r_p}{6(r_p + r_b)^5} \left[(2r_p - r_b) \left(\frac{\partial r_p}{\partial e_b} \right)^2 - r_p(r_p + r_b) \frac{\partial^2 r_p}{\partial e_b^2} \right] \end{aligned} \right\} \quad (3-41)$$

and

$$e = e_g + \frac{v_p}{\mu} \quad (3-42)$$

For common high-vacuum triodes, Eq. (3-40) converges rapidly enough so that the required accuracy is usually obtained with only a few terms of the series if the current amplitude is not too great. $\partial r_p / \partial e_b$ may be evaluated by plotting a curve of r_p vs. e_b determined from the static plate characteristic corresponding to the given operating grid bias. $\partial r_p / \partial e_b$ is the slope of this r_p - e_b curve at the point corresponding to the operating plate voltage. Other higher-order derivatives may be evaluated in a similar manner from curves of derivatives of the next lower order, but the accuracy rapidly decreases with increase of order of the derivative. The value of the series expansion lies not so much in the direct solution of numerical problems, as in the general analysis of the operation of vacuum tubes. In later chapters the series expansion will be applied to the analysis of the operation of vacuum tubes as amplifiers, detectors, modulators, and oscillators.

3-22. Derivation of Series Expansion.¹—Equations (3-40) and (3-41) are derived by applying Taylor's theorem for two variables to Eq. (3-2). This gives the following equation for alternating plate current:

$$i_p = \frac{1}{1!} \left(\frac{\partial i_b}{\partial e_g} e_g + \frac{\partial i_b}{\partial e_b} e_p \right) + \frac{1}{2!} \left(\frac{\partial i_b}{\partial e_g} e_g + \frac{\partial i_b}{\partial e_b} e_p \right)^2 + \frac{1}{3!} \left(\frac{\partial i_b}{\partial e_g} e_g + \frac{\partial i_b}{\partial e_b} e_p \right)^3 + \dots \quad (3-43)$$

in which a term of the form $\left(\frac{\partial i_b}{\partial e_g} e_g + \frac{\partial i_b}{\partial e_b} e_p \right)^n$ is a convenient symbolical representation of the terms²

¹ The following outline of the derivation of Eqs. (3-40) and (3-41) is included in order to explain the general method and justify the use of these equations throughout the remainder of the book. The student will not be seriously handicapped by omitting Sec. 3-22.

² Osgood, W. F., "Advanced Calculus," p. 172, The Macmillan Company, New York, 1925.